Parameter values $t \geq 3$ ($u_0 > 0$, $A = -3 - \ln 2$) may also be considered, corresponding to motion of the cylinder in the direction of the z axis in the presence of liquid injection through the surface.

NOTATION

x, y, Cartesian coordinates; r, z, cylindrical coordinates; r_0, cylinder radius; u, v, longitudinal and transverse (to the direction of surface motion) velocity components; u_0, velocity of surface motion; v_0, suction or injection rate through surface; q, parameter determining the suction or injection intensity; ν, kinematic viscosity; η, self-similar variable in Eq. (1); $\xi = r/r_0$, dimensionless radial coordinate; t, parameter in Eqs. (2) and (3) proportional to the boundary-layer thickness; δ, boundary-layer thickness; k, constant conditionally determining the boundary-layer thickness with respect to the degree of velocity drop at its boundary, $u = ku_0$ when $r = r_0 + \delta$; A, constant in Eqs. (2) and (3); z_*, combination of parameters with the dimensions of length, as defined in Eq. (2); α, auxiliary parameter defined in Eq. (2); $\overline{u} = u/u_0$, $\overline{v} = v/v_0$, $\overline{z} = z/|z_*|$, auxiliary parameters used in Figs. 2 and 3.

LITERATURE CITED

1. B. C. Sakiadis, AIChE J., 7, No. 1, 26-28 (1961).
2. B. C. Sakiadis, AIChE J., 7, No. 2, 221-225 (1961).
3. B. C. Sakiadis, AIChE J., 7, No. 3, 467-472 (1961).
4. E. A. Koldenhof, AIChE J., 9, No. 3, 411-418 (1963).
5. L. E. Erichson, L. T. Fan, and V. G. Fox, Ind. Eng. Ch. Fund., 5, No. 1, 19-25 (1966).

TURBULENT FLOW OF A FIBROUS SUSPENSION IN A PIPE

A. R. Kuznetsov, A. Ya. Ageev,
and S. A. Kuznetsova

UDC 532.542.4

The Henky–Ilyushin equations are used to describe the steady turbulent flow of an incompressible viscoplastic fluid in a pipe. A fibrous suspension is examined as the fluid.

The class of viscoplastic fluids contains a large number of systems such as cement mortars, oil–sand mixtures, oils, coal suspensions, etc. [1]. Fibrous suspensions of cellulose and asbestos in turbulent flow regimes can also be regarded as viscoplastic fluids.

A large number of investigations have been made of the laminar flow viscoplastic fluids, while the turbulent flow of these fluids has received little attention. Thus, the authors of the monograph [1] examined different problems connected mainly with the laminar motion of viscoplastic media. Several studies [2-4] have examined the laws governing the motion of fibrous suspensions. Here, researchers have obtained an extensive amount of experimental data and have developed an empirical approach to the study of the turbulent flow of fibrous suspensions. The authors of [5, 6] examined the turbulent flow of sand suspensions with the use of equations for each phase and with allowance for interaction of the phases. This approach leads to very complicated relations which include several unknowns and require time-consuming numerical study. In connection with this, it is interesting to examine the study [7]. Here, continuum conservation equations for the phases of the suspension were obtained using Feynman integrals over trajectories.

The motion of viscoplastic media is described by the Henky–Il'yushin differential equations. These equations appear as follows in vectorial form for an incompressible fluid [1]:

Lenin Komsomol Institute of Wood Technology, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 58, No. 2, pp. 223-229, February, 1990. Original article submitted October 6, 1988.

$$\frac{d\mathbf{u}}{dt}=\mathbf{F}-\frac{\operatorname{grad}p}{\rho}+\frac{1}{\rho}\left(\mu+\frac{\tau_0}{H}\right)\Delta\mathbf{u}-\frac{2\tau_0}{\rho H^2}S\operatorname{grad}H, \tag{1}$$

$$H=\left[\left(\frac{\partial u_1}{\partial y}+\frac{\partial u_2}{\partial x}\right)^2+\left(\frac{\partial u_1}{\partial z}+\frac{\partial u_3}{\partial x}\right)^2+\left(\frac{\partial u_2}{\partial z}+\frac{\partial u_3}{\partial y}\right)^2+\right.$$
$$\left.+2\left(\frac{\partial u_1}{\partial x}\right)^2+2\left(\frac{\partial u_2}{\partial y}\right)^2+2\left(\frac{\partial u_3}{\partial z}\right)^2\right]^{1/2}. \tag{2}$$

We will examine the turbulent steady flow of an incompressible viscoplastic fluid in a plane pipe. We direct the x axis of the Cartesian coordinate system parallel to the flow, while we place the y axis perpendicular to the broad walls of the pipe. Body forces will be ignored. Then we find from Eq. (1) that

$$\rho\frac{du_1}{dt}=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left[\left(\mu+\frac{\tau_0}{H}\right)\frac{\partial u_1}{\partial y}\right], \tag{3}$$

$$\frac{\partial p}{\partial y}=0, \tag{4}$$

$$\frac{\partial p}{\partial z}=0. \tag{5}$$

Averaging Eq. (3) over time with allowance for the mean and fluctuation values of velocity and pressure in the flow, we obtain the following (the bar above the quantities denotes averaging)

$$\frac{\partial p}{\partial x}=\frac{\partial}{\partial y}\left(\mu\frac{\partial u_1}{\partial y}+\tau_0-\rho\overline{u_1'u_2'}\right). \tag{6}$$

In the Boussinesq approximation [8], we write Eq. (6) in the form

$$\frac{\partial p}{\partial x}=\frac{\partial}{\partial y}\left[(\mu+A)\frac{\partial u_1}{\partial y}+\tau_0\right]. \tag{7}$$

Analyzing the differential equations of motion of the fluid in stresses [8] and Eq. (7), we can conclude that the expression in brackets in Eq. (7) is the component τ_{12} of the stress tensor:

$$\tau_{12}=(\mu+A)\frac{\partial u_1}{\partial y}+\tau_0. \tag{8}$$

As a rule, the quantity μ can be ignored. Thus, we find from (8) that

$$\tau_{12}=A\frac{\partial u_1}{\partial y}+\tau_0. \tag{9}$$

It is known [8] that the flow of a fluid in plane and circular pipes is described by the same relations. Thus, we can use Eq. (9) to describe the motion of a viscoplastic fluid in a circular pipe. This is of great practical importance, and the results obtained in this case would be important for understanding the laws governing flow under any other conditions [8].

We will examine the motion of a fibrous suspension in a circular pipe. A rod regime of flow is seen [2] for low mean flow velocities. An increase in velocity is accompanied by stratification of the dispersed suspension next to the pipe wall and retention of the rod regime in the center. The flow of the suspension is turbulent in this case [9]. It was termed transitional in [2]. Finally, at high flow velocities, the rod in the center of the pipe disintegrates and the suspension exhibits dispersed flow [2].

It should be noted that turbulence can arise in the rod flow regime for a fibrous suspension when the concentration is on the order of 1% (such concentrations are frequently used in practice, such as in the cellulose-paper industry) in the layer of water next to the pipe wall. This phenomenon has been observed for a wide range of pipe diameters [2, 3, 4, 9]. Thus, a turbulent regime is seen in transitional and dispersed flows of the suspension. In connection with this, we will not examine questions related to the transition to turbulence in the present study.

Using the Prandtl formula for eddy viscosity [8] and Eq. (9), we obtain

$$u=\frac{v_{*\mathrm{ef}}}{\varkappa_{\mathrm{n}}}\ln y+c,\quad u=u_1,\quad v_{*\mathrm{ef}}=\sqrt{\frac{\tau_w-\tau_0}{\rho}}. \tag{10}$$

We use (10) to find the following formula to calculate the velocity defect of the suspension

$$u_{\max}-u=\frac{v_{*\mathrm{ef}}}{\varkappa_{\mathrm{n}}}\ln\frac{R_{\mathrm{ef}}}{y},\quad R_{\mathrm{ef}}=R\left(1-\frac{\tau_0}{\tau_w}\right). \tag{11}$$

In the transitional and dispersed flows of a fibrous suspension with a concentration on the order of 1%, the concentration of the suspension next to the pipe wall is close to zero [9]. We will therefore assume that the distribution of dimensionless eddy viscosity $A/(v_*R\rho)$ along the dimensionless coordinate y/R_{ef} is close to linear and is independent of the Reynolds number. Nearly the same distribution is valid for water [8].

Having assumed, as in the case of water, that the eddy viscosity of the suspension at the distance y_0 from the wall is constant, we obtain

$$y_0=\frac{\nu\beta_{\mathrm{n\,ef}}}{v_*},\quad \beta_{\mathrm{n\,ef}}=\beta_{\mathrm{n}}\left(1-\frac{\tau_0}{\tau_w}\right),\quad v_*=\sqrt{\frac{\tau_w}{\rho}}. \tag{12}$$

With allowance for (12), we can use (10) to find a formula to calculate the velocity diagrams of a fibrous suspension in hydraulically smooth pipes

$$\frac{u}{v_*}=\frac{v_{*\mathrm{ef}}}{v_*\varkappa_{\mathrm{n}}}\ln\frac{v_*y}{\nu\beta_{\mathrm{n\,ef}}}+D_{\mathrm{n}}\quad D_{\mathrm{n}}=\frac{u_0}{v_*}. \tag{13}$$

We will examine the motion of a fibrous suspension in a hydraulically rough pipe. We write Eq. (13) in the following form:

$$\frac{u}{v_*}=\frac{v_{*\mathrm{ef}}}{v_*\varkappa_{\mathrm{n}}}\left(\ln\frac{y}{k_s}-\ln\frac{\beta_{\mathrm{n\,ef}}}{\alpha}\right)+D_{\mathrm{n}},\quad \alpha=\frac{v_*k_s}{\nu}. \tag{14}$$

We will assume that the following equality is valid under the condition $\tau_w \gg \tau_0$ in the intermediate regime and in the regime in which pipe roughness is manifest to the full extent

$$-\frac{1}{\varkappa_{\mathrm{n}}}\ln\frac{\beta_{\mathrm{n}}}{\alpha}+D'_{\mathrm{n}}=-\frac{1}{\varkappa_{\mathrm{n}}}\ln\frac{\beta_{\mathrm{n}}}{\alpha_{\mathrm{cr}}}+D_{\mathrm{n}}=\mathrm{const.} \tag{15}$$

In terms of structure, the quantity α is similar to the Reynolds number. It therefore has some constant value during the transition from laminar to turbulent flow.

We also note that assumption (15) was used in calculations for the flow of water [10]. In the general case, we find from (15) that

$$-\frac{v_{*\mathrm{ef}}}{v_*\varkappa_{\mathrm{n}}}\ln\frac{\beta_{\mathrm{n\,ef}}}{\alpha}+D'_{\mathrm{n}}=-\frac{v_{*\mathrm{ef}}}{v_*\varkappa_{\mathrm{n}}}\ln\frac{\beta_{\mathrm{n\,ef}}}{\alpha_{\mathrm{cr}}}+D_{\mathrm{n}}, \tag{16}$$

$$D'_{\mathrm{n}}=D_{\mathrm{n}}-\frac{v_{*\mathrm{ef}}}{v_*\varkappa_{\mathrm{ef}}}\ln\frac{\alpha}{\alpha_{\mathrm{ef}}}. \tag{17}$$

Replacing D_{n} by D'_{n} in the case of flow of the suspension in a rough pipe and allowing for (17), we can use (14) to obtain a formula for calculation of the velocity diagrams in rough pipes

$$\frac{u}{v_*}=\frac{v_{*\mathrm{ef}}}{v_*\varkappa_{\mathrm{n}}}\left(\ln\frac{y}{k_s}-\ln\frac{\beta_{\mathrm{n\,ef}}}{\alpha}-\ln\frac{\alpha}{\alpha_{\mathrm{cr}}}\right)+D_{\mathrm{n}}, \tag{18}$$

or

$$\frac{u}{v_*}=\frac{v_{*\mathrm{ef}}}{v_*\varkappa_{\mathrm{n}}}\ln\frac{y}{k_s\gamma_{\mathrm{ef}}}+D_{\mathrm{n}},\ \gamma_{\mathrm{ef}}=\gamma\left(1-\frac{\tau_0}{\tau_w}\right),\quad \gamma=\frac{\beta_{\mathrm{n}}}{\alpha_{\mathrm{cr}}}. \tag{19}$$

As the quantity α, the quantity β_{n} is similar in structure to the Reynolds number. Thus, the constant γ — being the ratio of two quantities of the same physical nature — is independent of the properties of the suspension and is roughly equal to the analogous constant for water ($\gamma \approx 70/5 = 14$).

At $\alpha = \alpha_{\mathrm{cr}}$, the pipe becomes hydraulically smooth and Eq. (19) becomes Eq. (13).

Thus, three different regimes can be distinguished for the turbulent pipe flow of a fibrous suspension:

1) regime in which roughness is not manifest, when

$$0 \leqslant \alpha \leqslant \alpha_{cr}; \tag{20}$$

2) intermediate regime

$$\alpha_{cr} \leqslant \alpha \leqslant \beta_{n_{ef}}; \tag{21}$$

3) regime in which roughness is fully manifest

$$\alpha > \beta_{n_{ef}}. \tag{22}$$

Let us deduce the laws governing resistance in the flow of a fibrous suspension in the turbulent regime [9]. During the transient motion of the suspension, a fibrous rod is present in the center of the flow. Using Eq. (11) for the velocity defect, we can determine the mean velocity over the annular cross section S_a in which the dispersed suspension is flowing:

$$\frac{u_{max} - u'_{av}}{v_{*\,ef}} = -\frac{1}{\varkappa_n S_a} \int_0^{R_{ef}} 2\pi (R - y) \left(\ln \frac{y}{R_{ef}} \right) dy =$$
$$= \frac{2\pi R_{ef}^2}{\varkappa_n S_a} \left(\frac{a}{R_{ef}} + \frac{3}{4} \right), \quad a = R - R_{ef}. \tag{23}$$

From here

$$u'_{av} = u_{max} - \frac{2\pi R_{ef}}{\varkappa_n S_a} \left(\frac{a}{R_{ef}} + \frac{3}{4} \right) v_{*ef}. \tag{24}$$

We find the average velocity over the entire cross section of the pipe S:

$$u_{av} = [u'_{av} S_a + u_{max} (S - S_a)]/S, \quad S = \pi R^2. \tag{25}$$

Having inserted (24) into (25), we obtain

$$u_{av} = u_{max} - \frac{2R_{ef}^2}{\varkappa_n R^2} \left(\frac{a}{R_{ef}} + \frac{3}{4} \right) v_{*ef}. \tag{26}$$

Expressing the quantity u_{max} in (26) by means of formulas (13) and (14) for the velocity distribution in smooth and rough pipes, we obtain the laws governing resistance for hydraulically smooth pipes

$$u_{av}(\tau_w) = \frac{v_{*ef}}{\varkappa_n} \ln \frac{v_* R}{\nu \beta_n} + D_c v_* - \frac{2(1 - \tau_0/\tau_w)^2}{\varkappa_n} [\tau_0/(\tau_w - \tau_0) + 3/4] v_{*ef} \tag{27}$$

and hydraulically rough pipes

$$u_{cp}(\tau_w) = \frac{v_{*ef}}{\varkappa_n} \ln \frac{R}{k_s \gamma} + D_n v_* - \frac{2(1 - \tau_0/\tau_w)^2}{\varkappa_n} [\tau_0/(\tau_w - \tau_0) + 3/4] v_{*ef}. \tag{28}$$

It should be noted that with a reduction in the concentration of the fibrous suspension to zero

$$\tau_0 \to 0, \ \varkappa_n \to \varkappa = 0.4, \ \beta_n \to \beta = 70,$$
$$\alpha_{av} \to \alpha_{cr.w} = 5, \ D_n \to D = 16.1 \tag{29}$$

and Eqs. (13), (19), (27), and (28) become the well-known formulas for water.

Let us examine some experimental results [9]. Figure 1 shows empirical data and theoretical flow curves (dashed lines) for an asbestos suspension in a pipe which is hydraulically smooth for the flow of a suspension with a concentration of 1.1 wt. % at $\tau_w \geq 9.5$ Pa (with allowance for Eq. (20) and the values $\beta_n = 80$, $\alpha_{cr} = 8$ obtained from experimental data). The figure also shows results of calculation of the resistance curve of a suspension with a concentration of 1.1 wt. % based on Eq. (27) for $\tau_w \leq 9.5$ Pa and based on Eq. (28) for $\tau_w \geq 9.5$ Pa. Here, we used constant values of the suspension parameters: $\tau_0 = 4$ Pa, $\kappa_n = 0.32$, $D_n = 21.3$ (these parameters were determined with the use of the experimental data in [9] and the relations obtained in the present investigation). It is evident that for values of the effective wall shear stress $\tau_{wef} = \tau_w - \tau_0$ on the order of 100 Pa, the empirical data deviates from the theoretical results and approaches the resistance curve. This is

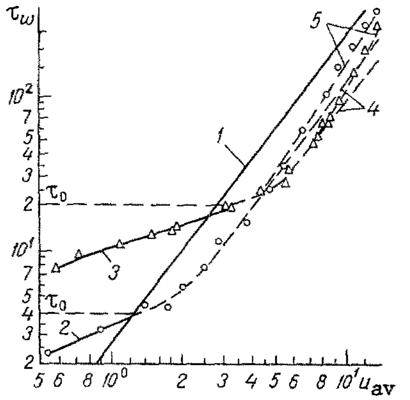

Fig. 1

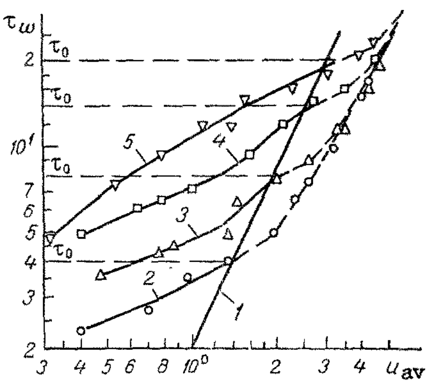

Fig. 2

Fig. 1. Experimental data (solid lines and points) and theoretical curves (dashed) describing the resistance encountered in the flow of an asbestos suspension in a pipe D_p = 98 mm with a roughness $k_s/D_p = 8.4 \cdot 10^{-4}$: 1) for water; 2) concentration of suspension 1.1 wt. %; 3) 2.4 wt. %; 4) calculation with constant κ_n, D_n; 5) calculation with variable κ_n, D_n, τ_w, Pa; u_{av}, m/sec; τ_0, Pa.

Fig. 2. Experimental data (solid curves and points) and theoretical curves (dashed) describing the resistance encountered in the motion of an asbestos suspension in a pipe D_p = 213 mm with a roughness $k_s/D_p = 1.8 \cdot 10^{-4}$; 1) for water; 2) concentration of suspension 1.1 wt. %; 3) 1.5 wt. %; 4) 2.0 wt. %; 5) 2.4 wt. %.

evidently due to the fact that the extinction of turbulent pulsations in the flow by the fibers diminishes with an increase in $\tau_{w_{ef}}$. Here, the values of the parameters of the suspension κ_n, β_n, α_{cr}, and D_n approach the analogous constants for water and, in the limit, the resistance curves of the suspension and water merge.

Analysis of the experimental data in [9] showed that the following relation exists for the parameters of the suspension:

$$p_n' = p_{\hat{n}} + (p_w - p_n)[1 - \exp(-k_n \tau_{w_{ef}})]. \tag{30}$$

The coefficient k_n can be taken to be constant for all of the parameters. For an asbestos suspension with a concentration of 1.1 wt. %, $k_n = 4.4 \cdot 10^{-3}$ 1/Pa.

It is evident from Fig. 1 that in the case of variable parameters, the theoretical resistance curve of the suspension satisfactorily agrees with the experimental data.

Figure 2 shows that satisfactory agreement is also obtained between the empirical data and data calculated from Eq. (27) for the motion of an asbestos suspension in a hydraulically smooth pipe.

It is evident from Figs. 1 and 2 that the limiting shear stresses of the suspension correspond approximately to the points of intersection of the resistance curves for the suspension and water and increase with an increase in the concentration of the suspension.

Figure 3 shows theoretical resistance curves for the motion of an asbestos suspension in a pipe in the case where the roughness of the pipe wall becomes manifest.

Thus, study of the steady turbulent flow of an incompressible viscoplastic fluid in a pipe has yielded relations which describe the turbulent flow of a fibrous suspension in pipes. These relations can be used to calculate the hydraulic transport of suspensions and to design hydraulic machinery and equipment in which the working fluid is a suspension.

NOTATION

$\mathbf{u}$, flow velocity vector; u_1, u_2, u_3, projections of the velocity vector on the coordinate axes; u_2', u_1', fluctuation velocities; t, time; $\mathbf{F}$, vector of body forces; ρ, density of suspen-

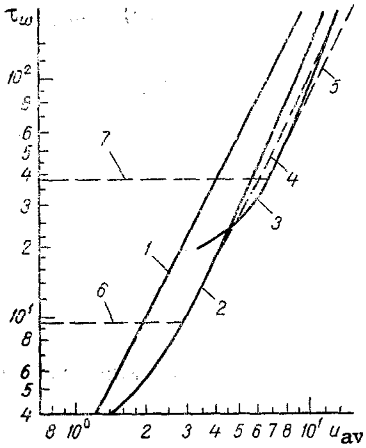

Fig. 3. Theoretical resistance curves for the motion of an asbestos suspension in a pipe D_p = 98 mm with different wall roughnesses: 1) for water; 2, 3) for a suspensions with a concentration of 1.1 and 2.4 wt. %, respectively, at $k_s/D_p = 8.4 \cdot 10^{-4}$; 4, 5) same with $k_s/D_p = 0$; 6, 7) critical values of τ_w corresponding to the transition from the flow regime for hydraulically smooth pipes to the regime for rough pipes for curves 2 and 3.

sion; p, pressure; μ, ν, dynamic and kinematic viscosities of the dispersion medium; τ_0, limiting shear stress; H, intensity of shear strain rate; A, eddy viscosity, or coefficient of turbulent exchange; τ_{12}, component of stress tensor; τ_w, $\tau_{w_{ef}}$, shear stress and effective shear stress on the pipe wall; v_*, v_{*ef}, dynamic and effective dynamic velocity; R, R_{ef}, radius and effective radius of pipe; u_{max}, maximum velocity; c, constant of integration in (10); κ, β, D, $\alpha_{cr.w}$, constants of water; κ_n, β_n, D_n, α_{cr}, parameters of the suspension dependent on its properties (concentration, type of fibers, etc.); y_0, distance from wall to boundary of turbulent core of flow; $\beta_{n_{ef}}$, effective value of β_n; α, parameter; u_0, velocity next to the boundary of the turbulent core of the flow; k_s, equivalent sandy roughness of pipe wall; D_n', parameter of the suspension characterizing its flow in a hydraulically rough pipe; γ, γ_{ef}, constant in (19) and its effective value; a, radius of fibrous rod in the center of the flow; S, cross-sectional area of pipe; S_a, area of annular cross section of pipe in which the dispersed suspension flows; D_p, diameter of pipe; p_n, values of the suspension parameters κ_n, β_n, D_n with a value of $\tau_{w_{ef}}$ close to zero; p_n', values of the suspension parameters κ_n, β_n, D_n with the given value of $\tau_{w_{ef}}$; p_w, values of the constants κ, β, D for water; k_n, coefficient; u_{av}', mean velocity in the annular section of the channel S_a; u_{av}, mean velocity in the pipe; S, tensor of shear strain rate.

LITERATURE CITED

1. P. M. Ogibalov and A. Kh. Mirzadzhanzade, Transient Motions of Viscoplastic Media [in Russian], Moscow (1977).
2. O. A. Terent'ev, Hydrodynamics of Fibrous Suspensions in the Cellulose-Paper Industry [in Russian], Moscow (1980).
3. V. I. Klimov, Design and Planning of Product Pipelines in the Cellulose-Paper Industry [in Russian], Leningrad (1986).
4. W. Mih and J. Parker, TAPPI, 50, No. 5, 237-246 (1967).
5. M. C. Roco and N. Balakrishnam, J. Fluids, 29, No. 4, 431-456 (1985).
6. M. C. Roco and C. A. Shook, J. Fluids Eng., 107, No. 6, 224-231, 107, (1985).
7. V. S. Belousov, Yu. A. Buevich, and G. P. Yasnikov, Inzh. Fiz.-Zh., 48, No. 4, 602-609 (1985).
8. H. Schlichting, Boundary Layer Theory, McGraw-Hill, New York (1968).
9. A. R. Kuznetsov, Optimization of the Casting of Asbestos-Fiber Sheets on Papermaking Machines: Author's Abstract of Engineering Sciences Candidate Dissertation, Leningrad (1986).
10. A. D. Al'tshul', Hydraulic Resistances, Moscow (1970).